

Theory of a.c. spin current noise and spin conductance through a quantum dot in the Kondo regime I: The equilibrium case

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We analyze the equilibrium frequency-dependent spin current noise and spin conductance through a quantum dot in the local moment regime. Spin current correlations behave markedly differently from charge correlations. Equilibrium spin correlations are characterized by two universal scaling functions in the absence of an external field: one of them is related to charge correlations, while the other one describes cross-spin correlations. We characterize these functions using a combination of perturbative and non-perturbative methods. We find that at low temperatures spin cross-correlations are suppressed at frequencies below the Kondo scale, T_K , and a dynamical spin accumulation resonance is found at the Kondo energy, $\omega \sim T_K$. At higher temperatures, $T > T_K$, surprising low-frequency anomalies related to overall spin conservation appear in the spin noise and spin conductance, and the Korringa rate is shown to play a distinguished role. The transient spin current response also displays universal and singular properties.

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I. INTRODUCTION

Nanoelectronic devices are likely to provide our future technology and serve as basic tools for storing information, quantum computation^{1,2} or spin manipulation.^{3,4} Due to recent developments in fabrication, it is now possible to produce and measure spin accumulation in mesoscopic circuits, or filter the generated spin currents.⁵⁻⁸ One of the next most prominent goals of spintronics is to go further towards the microscopic regime,⁹⁻¹¹ and try to measure and manipulate *single spins* in quantum dots using spin biased circuits. Understanding the structure of spin current noise and response through quantum dots is therefore of primary importance. Moreover, the interplay of strong interactions and the impact of quantum fluctuations of the spin on spin transport are also important questions of fundamental interest on their own, and quantum dots, being the simplest strongly interacting systems, also play a prominent role in this regard: they allow for the systematic and controlled experimental and theoretical study of strongly interacting states. Although not easy to measure,¹²⁻¹⁷ dynamical noise spectra and response functions would allow to gain a clear insight to the structure of interactions.¹⁸

In spite of its obvious importance, however, surprisingly little is known about the dynamical spin current response and the spin current noise spectrum of a quantum dot. Spin current shot noise in the sequential tunneling regime has been theoretically studied in Ref. [19], and later in the co-tunneling regime by Kindermann.²⁰ However, these calculations focussed almost exclusively on d.c. properties, and have avoided the strong coupling (Kondo) regime,²¹ which is much more difficult to reach

and understand theoretically.

In the present paper we make an attempt to characterize the equilibrium spin noise spectrum and the dynamical spin response of a quantum dot. Here we focus on the local moment regime,²² where charge fluctuations can be neglected, and the dot can simply be described in terms of a local spin operator, \mathbf{S} ($S = 1/2$), coupled to the left and right lead electrons through an exchange coupling, j . We shall study how time dependent spin polarized currents can be injected through the dot at various temperatures, and how these spin polarized currents fluctuate in time. To obtain a coherent and clear picture, we combine various numerical and analytical methods such as numerical renormalization group (NRG), perturbative and renormalization group calculations, and a master equation approach.

Though some of the results presented here are also valid in non-equilibrium, in this paper we focus exclusively on the equilibrium case, and leave the detailed presentation of the rather technical non-equilibrium quantum Langevin calculation to a subsequent publication.²³ Nevertheless, even in this equilibrium case, the spin noise and the spin conductance display an extremely rich structure; in addition to the temperature, T , and the Kondo temperature, T_K , below which the dot spin gets screened, new time scales emerge. In the regime, $T \gg T_K$, e.g., we find that the Korringa rate (the characteristic spin decay rate),

$$E_K \approx \frac{\pi T}{\ln^2(T/T_K)}, \quad (1)$$

plays a distinguished role. Furthermore, incorporating the effect of external spin relaxation processes also turns

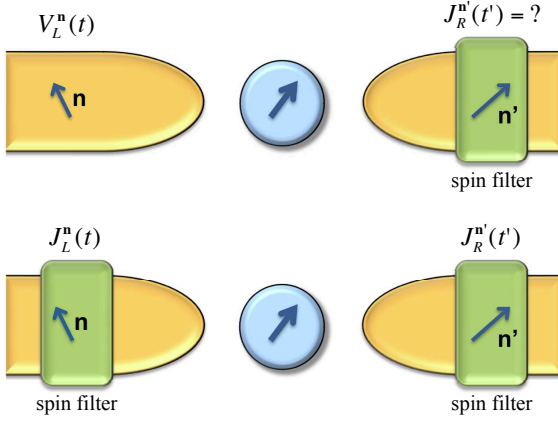


FIG. 1: (Color online) Sketch of the setups to measure the spin-dependent left-right conductance, $G_{LR}^{nn'}(\omega)$, and the noise, $S_{LR}^{nn'}(\omega)$. The arrows indicate the direction of the spin polarization in each lead. In the upper setup the spin filter on the right side detects the spin-resolved current at time t' , $J_R^{n'}(t')$, induced by applying a spin-dependent voltage to the left lead at time t , $V_L^n(t)$. The lower setup allows for injecting spins and measuring the spin-resolved currents in both electrodes.

out to be important, and introduces an additional new rate, $1/\tau_s$.

In our analysis, we focus on two crucial quantities. On the one hand, we study the dynamical spin conductance $G_{rr'}^{nn'}(\omega)$. This quantity characterizes how a current $J_r^n(t)$ of carriers with spins polarized in direction \mathbf{n} is generated in the left ($r = L$) or right ($r = R$) lead by a time dependent chemical potential shift, $\delta\mu_{r'}^{n'}(t) = eV_{r'}^{n'}(t)$, acting in lead r' on carriers polarized along \mathbf{n}' (see Fig. 1),

$$\langle J_r^n(t) \rangle = \int dt' G_{rr'}^{nn'}(t - t') V_{r'}^{n'}(t'). \quad (2)$$

Furthermore, we also investigate time dependent correlations of the currents $J_r^n(t)$,

$$S_{rr'}^{nn'}(t - t') \equiv \frac{1}{2} \langle \{ J_r^n(t), J_{r'}^{n'}(t') \} \rangle, \quad (3)$$

and determine the corresponding noise spectra. Of course, in the equilibrium case studied here, $G_{rr'}^{nn'}$ and $S_{rr'}^{nn'}$ are not independent, but are related by the fluctuation dissipation theorem [see Eq. (26)].

One of our main results is, that - in the absence of external spin relaxation and external magnetic field - both the noise and the conductance take on simple, universal forms, and apart from some geometry dependent prefactors, are characterized by just two universal functions. The left-right ($r = L$, $r' = R$) conductance, e.g., for $\mathbf{n} = \sigma\hat{z}$ and $\mathbf{n}' = \sigma'\hat{z}$ reads

$$G_{LR}^{\sigma\sigma'}(\omega) = \frac{e^2}{h} \sin^2(\phi) [\delta_{\sigma\sigma'} \tilde{g}(\omega, T) + \sigma\sigma' g(\omega, T)], \quad (4)$$

where only the prefactor depends on the specific geometry of the dot, characterized by the angle ϕ (see Sec. II

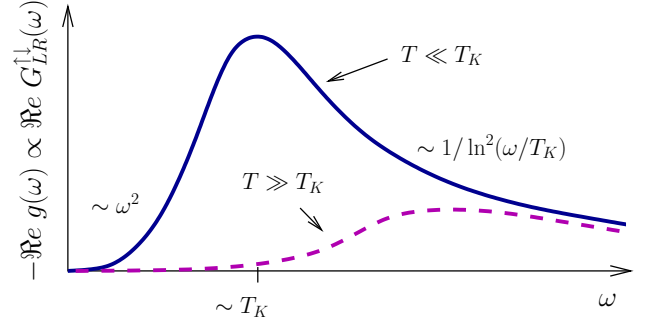


FIG. 2: (Color online) Sketch of the properties of the real part of the universal function, $-g(\omega, T) \sim G_{LR}^{\uparrow\downarrow}(\omega)$, characterizing the cross-spin conductance through the dot.

and Ref. [24]), but the functions g and \tilde{g} are universal functions of ω/T_K and T/T_K . The prefactor, $\frac{e^2}{h}$ in Eq. (4), denotes the universal conductance quantum.⁴⁹ The function \tilde{g} describes charge conductance through the dot, while g determines the cross-spin ($\sigma = \uparrow$, $\sigma' = \downarrow$) conductance, $G_{rr'}^{\uparrow\downarrow}(\omega)$. Our main goal is to study the properties of g and those of the corresponding noise component in detail. The characteristic features of g are shown in Fig. 2, which gives a concise summary of our most important results. As shown in Fig. 2, g vanishes in the limit, $\omega \rightarrow 0$, and $|g|$ develops a dip at $\omega < T_K$, and a broad resonance at $\omega \sim T_K$ for temperatures $T \ll T_K$. The vanishing of the d.c. conductance is a consequence of the fact that spin can only be transferred from the spin-up channel to the spin-down channel by flipping the dot spin, S . Thus the amount of spin transfer is limited, and no d.c. spin-cross conductance is possible in the absence of external spin relaxation. Temporarily, however, one can transfer spin between these two channels at the expense of accumulating spin on the dot. This amounts in the appearance of the broad resonance.

The above-mentioned dip in $|\Re g|$ also survives at temperatures, $T \gg T_K$: There, by the simple argument above, the cross-spin conductance also goes to zero

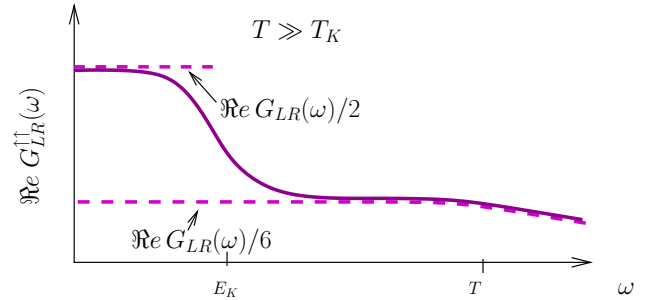


FIG. 3: (Color online) Real part of the conductance $G_{LR}^{\uparrow\downarrow}(\omega)$ through the dot for $T \gg T_K$. The anomaly below the Korrington relaxation rate, E_K , is a consequence of correlations between consecutive spin-flip processes, and is related to the dip in $|\Re g(\omega)|$ (see Fig. 2).

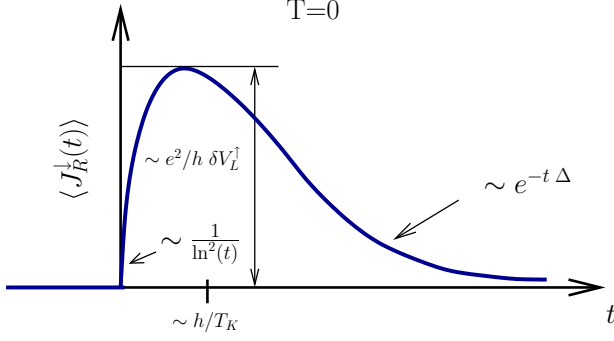


FIG. 4: (Color online) Real time response of the current $\langle J_R^\downarrow(t) \rangle$, as generated by a sudden change of amplitude, δV_L^\uparrow .

as $\omega \rightarrow 0$, however, this happens only below the Korrington rate, $\omega < E_K$ [see Eq. (1)], where correlations between consecutive spin-flip events become important. As a consequence of this, the spin-up – spin-up conductance, $G_{LR}^{\uparrow\uparrow}(\omega)$, develops a *peak* for frequencies $\omega < E_K$. The relative size of the peak is universal (see Fig. 3), and is determined by the ratio of spin-flip in the high temperature regime vs spin-diagonal scattering processes.

By just analyzing the analytical structure of the conductance, $G_{rr'}^{\sigma\sigma'}(\omega)$, we can also make rather general conclusions on the structure of transient response to a sudden potential change at time $t = 0$, $V_r^\sigma(t) = \delta V_r^\sigma \Theta(t)$, where $\Theta(t)$ denotes the step function. The linear response is universal, and it depends only on tT_K and T/T_K . Figure 4 sketches the structure of the zero-temperature response, $\langle J_R^\downarrow(t) \rangle$ generated by a sudden change of V_L^\uparrow . The response is found to be logarithmically singular at time $t = 0$, and at $T = 0$ temperature, one observes at a time scale $t \approx h/T_K$ an induced current bump of amplitude $\sim (e^2/h) \delta V_L^\uparrow$, followed by an exponential decay of the current response. The exponential decay we find is somewhat counter-intuitive: the long-time behavior is typically associated with Fermi liquid properties,²⁵ and therefore one could naively expect an algebraic decay. However, the exponential decay found follows precisely from Fermi liquid theory, which implies certain analytical properties for the response functions.

As mentioned before, the properties of the equilibrium noise spectrum are related to those of the conductance.²⁶ Thus the noise has a structure similar to Eq. (4), and can also be described in terms of just two universal functions, $\tilde{s}(\omega)$ and $s(\omega)$, describing charge and cross-spin correlations, respectively. The left-right noise components $S_{LR}^{\sigma\sigma'}(\omega)$, e.g., can be expressed as

$$S_{LR}^{\sigma\sigma'}(\omega) = -\frac{e^2}{h} T_K \sin^2(\phi) [\delta_{\sigma\sigma'} \tilde{s}(\omega, T) + \sigma\sigma' s(\omega, T)] , \quad (5)$$

with the two universal functions \tilde{s} and s displaying markedly different behavior. At $T \rightarrow 0$, e.g., $\tilde{s}(\omega) \sim |\omega|/T_K$, while spin cross-correlations scale as $s(\omega) \sim |\omega|^3/T_K^3$.

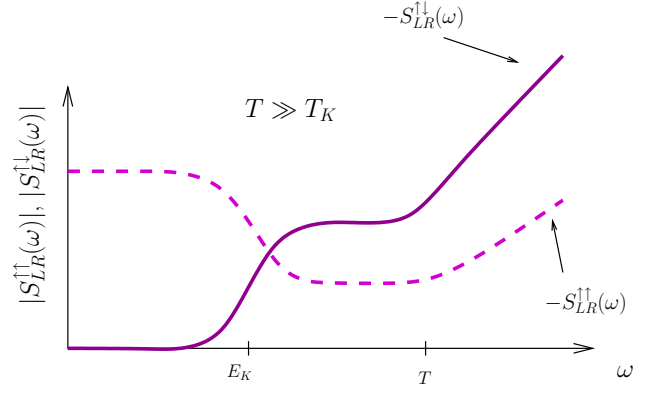


FIG. 5: (Color online) Structure of the spin noise components $S_{LR}^{\uparrow\uparrow}(\omega)$ and $S_{LR}^{\downarrow\downarrow}(\omega)$ for $T \gg T_K$.

This difference between s and \tilde{s} carries over to finite temperatures, where \tilde{s} remains finite in the d.c. limit, while the spin-dependent component s of the noise always scales to zero as $\omega \rightarrow 0$. In particular, at temperatures $T \gg T_K$ the charge component of the noise is similar to what is found for an ordinary tunnel junction,^{27,28}

$$\tilde{s}(\omega, T \gg T_K) \approx \frac{3\pi^2}{16} j^2(\omega, T) \frac{\omega}{T_K} \coth\left(\frac{\omega}{2T}\right) , \quad (6)$$

and remains proportional to the temperature, T , for $\omega \rightarrow 0$. Here the only nontrivial physics is carried by the prefactor $j^2(\omega, T) \approx 1/\ln^2(\max\{|\omega|, T\}/T_K)$, which accounts for the Kondo-renormalized amplitude of the current, mediated by the exchange processes. Similar to g , however, the spin-dependent part s has a non-trivial structure, and exhibits a dip at frequencies below the Korrington rate, $\omega < E_K$, where correlations between consecutive spin-flip processes drive the cross-spin noise to zero. This amounts in somewhat unexpected low frequency anomalies in the noise spectra $S_{rr'}^{\sigma\sigma'}(\omega)$, as sketched in Fig. 5 for the particular case of the left-right noise components.

The results discussed so far hold in the case, where spin relaxation on the dot is induced exclusively by the exchange coupling, j , to the lead electrons. Then the d.c. cross-spin conductance and the shot noise vanish due to spin conservation. External spin relaxation, however, slightly modifies the picture above, and in its presence we find a small but finite spin shot noise and d.c. spin conductance. The precise dependence of the noise on the spin relaxation rate, $1/\tau_s$, is rather complex, and is analyzed in Sec. VI in detail. Experimentally, however, the most relevant regime seems to be the one, where $h/\tau_s \ll T, E_K$. There we find a residual cross-spin noise

$$S_{LR}^{\uparrow\downarrow}(0) \sim \begin{cases} -\frac{e^2}{h} \frac{h}{\tau_s}, & \text{if } T \gg T_K , \\ -\frac{e^2}{h} T \left(\frac{h}{\tau_s T_K}\right)^2, & \text{if } T \ll T_K . \end{cases} \quad (7)$$

For typical quantum dots, this residual noise and the corresponding conductance are thus very small compared

to all other features, and therefore inclusion of $1/\tau_s$ only slightly modifies the picture obtained within the Kondo model.

The paper is organized as follows. In Sec. II we describe the system's Hamiltonian and define the spin current operators. We then derive compact expressions for the spin noise and spin conductance in terms of universal functions using the Kubo formula. The zero temperature behavior is discussed in Sec. III, where we first show the analytical results in the perturbative and Fermi liquid regimes, and then present NRG results for universal functions obtained in the absence/presence of external magnetic field. As shown in Section III.B, application of an external magnetic field leads to a more complex behavior and has somewhat similar effect as the external spin relaxation for spin currents polarized perpendicular to the applied field.

Section IV is devoted to the study of the role of a finite temperature. In the perturbative regime, to determine the ω -dependence of universal functions we use a combination of master equation and perturbation theory, while in the strong coupling regime we resort to the Fermi liquid arguments. Transient response of the dot to a sudden switch of a time-dependent spin-resolved voltage is discussed in Sec. VI, while in Sec. VII we consider the effect of external spin relaxation on the dynamics of spin correlations. Finally, the conclusions are given in Sec. VII.

II. THEORETICAL FRAMEWORK

A. Hamiltonian

Throughout this paper, we focus our attention on the local moment regime of a quantum dot, where the dot is occupied by a single electron and it can be described in terms of a spin \mathbf{S} , coupled to the leads. The Hamiltonian of the system consists of a part describing electrons in the leads, H_{leads} , and an interacting part, H_{int} , which accounts for the exchange coupling between the dot and the leads, $H = H_{\text{leads}} + H_{\text{int}}$. As usual, electrons in the leads are assumed to be non-interacting, and in the absence of a charge or spin bias, they are described by the Hamiltonian,

$$H_{\text{leads}} = \sum_{r,\sigma} \int_{-D}^D \varepsilon c_{r\sigma}^\dagger(\varepsilon) c_{r\sigma}(\varepsilon) d\varepsilon, \quad (8)$$

with $c_{r\sigma}^\dagger(\varepsilon)$ being the creation operator for a spin- σ electron with energy ε in the left ($r = L$) or right ($r = R$) electrode and $2D$ being the bandwidth. The energy ε is measured from the chemical potential, and the operators $c_{r\sigma}(\varepsilon)$ satisfy the usual anticommutation relations: $\{c_{r\sigma}^\dagger(\varepsilon), c_{r'\sigma'}(\varepsilon')\} = \delta_{rr'} \delta_{\sigma\sigma'} \delta(\varepsilon - \varepsilon')$.⁵⁰

The interaction part, H_{int} , can be most easily constructed in terms of the fields, $\psi_{r\sigma} = \int_{-D}^D c_{r\sigma}(\varepsilon) d\varepsilon$, which

destroy electrons of spin σ in the lead r . In terms of these fields, the Kondo interaction is given by

$$H_{\text{int}} = \sum_{r,r'=L,R} \sum_{\sigma,\sigma'} \frac{j}{2} v_r v_{r'} \mathbf{S} \psi_{r\sigma}^\dagger \boldsymbol{\sigma}_{\sigma\sigma'} \psi_{r'\sigma'}, \quad (9)$$

with j a dimensionless coupling related to the Kondo temperature, $T_K \approx D e^{-1/j}$, and $\boldsymbol{\sigma}$ the Pauli spin matrices. Here the strength of the hybridization of the dot level with the leads is incorporated in the exchange coupling, j , while its left-right asymmetry appears through the dimensionless parameters, $v_{L/R}$. These satisfy the relation $v_L^2 + v_R^2 = 1$, and are parametrized by an angle ϕ as $v_L = \cos(\phi/2)$, $v_R = \sin(\phi/2)$.⁵¹ Using this parametrization the maximal, $T = 0$ temperature conductance is given by, $G_0 = (2e^2/h) \sin^2(\phi)$, where e^2/h is the conductance quantum. In the equilibrium case, studied here, the problem can be further simplified by noticing that only the combination $\Psi \equiv v_L \psi_L + v_R \psi_R$ occurs in Eq. (9). Therefore, rewriting the Hamiltonian in terms of the 'even' (Ψ) and 'odd' ($\tilde{\Psi}$) fields,

$$\begin{aligned} \Psi_\sigma &\equiv \cos\left(\frac{\phi}{2}\right) \psi_{L\sigma} + \sin\left(\frac{\phi}{2}\right) \psi_{R\sigma} \\ \tilde{\Psi}_\sigma &\equiv \sin\left(\frac{\phi}{2}\right) \psi_{L\sigma} - \cos\left(\frac{\phi}{2}\right) \psi_{R\sigma}, \end{aligned} \quad (10)$$

the interaction part simplifies to

$$H_{\text{int}} = \frac{j}{2} \mathbf{S} (\Psi^\dagger \boldsymbol{\sigma} \Psi), \quad (11)$$

and the 'odd' field completely decouples from the dot.

B. Spin current, spin conductance, and spin noise

To calculate the spin noise and the spin conductance, we first need to define the spin current operators. Let us first focus on the case, where only the z -component is measured, and define the operators,

$$Q_r^\sigma = e \int_{-D}^D c_{r\sigma}^\dagger(\varepsilon) c_{r\sigma}(\varepsilon) d\varepsilon, \quad (12)$$

measuring the total charge of the electrons in lead r and spin component, σ , where e is the electron charge. The corresponding current operator can then be defined as

$$J_r^\sigma \equiv \frac{dQ_r^\sigma}{dt}. \quad (13)$$

Notice that with this definition a positive current implies a charge flow towards the leads. Using the equation of motion we obtain

$$J_r^\sigma = ie \frac{j}{2} \sum_{r',\sigma'} v_r v_{r'} \mathbf{S} (\psi_{r\sigma}^\dagger \boldsymbol{\sigma}_{\sigma\sigma'} \psi_{r'\sigma'} - \text{h.c.}) . \quad (14)$$

The charge current is then simply expressed as the sum of spin currents, $J_r = \sum_\sigma J_r^\sigma$.

At this point, it is useful to also express the current operators in terms of the even and odd fields, Ψ and $\tilde{\Psi}$. Introducing the so-called composite fermion operators,²⁹ $F_\sigma \equiv (\mathbf{S} \boldsymbol{\sigma} \Psi)_\sigma$, we can write the current operator as a sum of two components,

$$\begin{aligned} J_r^\sigma &= I_r^\sigma + \tilde{I}_r^\sigma, \\ I_r^\sigma &= e j \gamma_r i(\Psi_\sigma^\dagger F_\sigma - F_\sigma^\dagger \Psi_\sigma), \\ \tilde{I}_r^\sigma &= e j \tilde{\gamma}_r i(\tilde{\Psi}_\sigma^\dagger F_\sigma - F_\sigma^\dagger \tilde{\Psi}_\sigma), \end{aligned} \quad (15)$$

with the prefactors defined as $\gamma_{L/R} = (1 \pm \cos(\phi))/4$, and $\tilde{\gamma}_{L/R} = \pm \sin(\phi)/4$.

It is instructive to analyze the properties of the current operators, Eq. (15). The components I_r^σ satisfy

$$I_r^\uparrow + I_r^\downarrow \equiv 0, \quad (16)$$

implying that the components I_r^σ do not contribute to the charge current, $J_r = \sum_\sigma \tilde{I}_r^\sigma$. Furthermore, since the components \tilde{I}_r^σ , satisfy current conservation at the operator level,

$$\tilde{I}_L^\sigma + \tilde{I}_R^\sigma \equiv 0, \quad (17)$$

there is no charge accumulation on the dot, and $J_L(t) + J_R(t) = 0$ is satisfied at the operator level at any time t . The current components, I_r^σ , on the other hand, do not satisfy current conservation, $I_L^\sigma + I_R^\sigma \neq 0$, implying that spin accumulation is possible on the dot. These properties follow naturally in the local moment (Kondo) limit, where charge fluctuations are completely suppressed, and only spin fluctuations are allowed on the dot.

C. Conductance and noise

We shall now analyze the structure of the current response of the dot generated by an external perturbation, $\delta H = V_r^\sigma(t) Q_r^\sigma(t)$, which shifts the potential of carriers with spin σ in lead r . In linear response, the current is determined by the conductance $G_{rr'}^{\sigma\sigma'}$ in Eq. (2), given by the Kubo formula,

$$G_{rr'}^{\sigma\sigma'}(t, t') = -i\Theta(t - t') \langle [J_r^\sigma(t), Q_{r'}^{\sigma'}(t')] \rangle. \quad (18)$$

This expression still contains the charge operators, which are non-local operators. To eliminate them, we differentiate with respect to time t' and use the equation of motion. After Fourier transformation we then obtain the following form of the Kubo formula,

$$i\omega G_{rr'}^{\sigma\sigma'}(\omega) = (J_r^\sigma; J_{r'}^{\sigma'})_\omega - (J_r^\sigma; J_{r'}^{\sigma'})_{\omega=0}, \quad (19)$$

where we introduced a compact notation for the Fourier transform of the retarded correlation function of any two operators,

$$\mathcal{G}_{AB}^R(\omega) \rightarrow (A; B)_\omega. \quad (20)$$

The second term in Eq. (19) originates from the discontinuity of $G_{rr'}^{\sigma\sigma'}(t)$ at $t = 0$, which can be shown to be real and just equal to $(J_r^\sigma; J_{r'}^{\sigma'})_{\omega=0}$.

Let us now focus on the $SU(2)$ symmetrical case. To simplify the expression of $G_{rr'}^{\sigma\sigma'}$, we can make use of the decomposition of I_r^σ , Eqs. (15), and exploit the property (16) as well as the fact that all cross-correlations of I_r^σ and $\tilde{I}_{r'}^{\sigma'}$ vanish. Taking furthermore the spin symmetry into account we find that $G_{rr'}^{\sigma\sigma'}$ reduces to the following form,

$$G_{rr'}^{\sigma\sigma'}(\omega) = \frac{e^2}{h} [\delta_{\sigma\sigma'} \tilde{a}_{rr'} \tilde{g}(\omega, T) + \sigma\sigma' a_{rr'} g(\omega, T)]. \quad (21)$$

Here the information on the geometry of the dot is exclusively carried by the matrices $\tilde{\mathbf{a}}(\phi)$ and $\mathbf{a}(\phi)$,

$$\tilde{\mathbf{a}} = \begin{pmatrix} -\sin^2 \phi & \sin^2 \phi \\ \sin^2 \phi & -\sin^2 \phi \end{pmatrix}, \quad \mathbf{a} = \begin{pmatrix} 4 \cos^2 \frac{\phi}{2} & \sin^2 \phi \\ \sin^2 \phi & 4 \sin^2 \frac{\phi}{2} \end{pmatrix},$$

while the functions g and \tilde{g} are completely independent of the dot geometry. They are defined in terms of the 'reduced current operators',

$$\mathcal{I}_\sigma \equiv i(\Psi_\sigma^\dagger F_\sigma - \text{h.c.}) \quad (22)$$

$$\tilde{\mathcal{I}}_\sigma \equiv i(\tilde{\Psi}_\sigma^\dagger F_\sigma - \text{h.c.}) \quad (23)$$

and are given by the following expressions,

$$\begin{aligned} g(\omega) &= +\frac{\pi j^2}{8i\omega} [(\mathcal{I}_\uparrow; \mathcal{I}_\uparrow)_\omega - (\mathcal{I}_\uparrow; \mathcal{I}_\uparrow)_{\omega=0}], \\ \tilde{g}(\omega) &= -\frac{\pi j^2}{8i\omega} [(\tilde{\mathcal{I}}_\uparrow; \tilde{\mathcal{I}}_\uparrow)_\omega - (\tilde{\mathcal{I}}_\uparrow; \tilde{\mathcal{I}}_\uparrow)_{\omega=0}]. \end{aligned} \quad (24)$$

Here we have chosen a normalization such that for the ordinary conductance through the dot we recover the well-known expression,

$$G_{LR}(\omega, T) = 2 \frac{e^2}{h} \sin^2(\phi) \tilde{g}(\omega, T), \quad (25)$$

with $\tilde{g}(\omega, T \rightarrow 0) = 1$ in the unitary limit.

Importantly, both $g(\omega, T)$ and $\tilde{g}(\omega, T)$ are dimensionless functions, and, apart from some universal prefactors, they both determine physically measurable quantities. By the basic principles of the renormalization group, this immediately implies that in the scaling limit, $j \rightarrow 0$, $D \rightarrow \infty$, and T_K finite, they must both reduce to functions of the ratios, ω/T_K and T/T_K . This can, of course, be checked explicitly by performing perturbation theory in the exchange coupling, j .

In the equilibrium case, to which we restricted ourselves here, the symmetrized current noise [Eq. (3)] is simply related to the retarded Green's functions, $(J_r^\sigma; J_{r'}^{\sigma'})_\omega$, and thus to the conductance by the fluctuation-dissipation theorem,

$$S_{rr'}^{\sigma\sigma'}(\omega) = -\omega \coth\left(\frac{\omega}{2T}\right) \Re G_{rr'}^{\sigma\sigma'}(\omega). \quad (26)$$

Using Eq. (21), we then immediately find

$$S_{rr'}^{\sigma\sigma'}(\omega) = -\frac{e^2}{h} T_K [\delta_{\sigma\sigma'} \tilde{a}_{rr'} \tilde{s}(\omega, T) + \sigma\sigma' a_{rr'} s(\omega, T)] ,$$

where s and \tilde{s} are two real dimensionless universal functions characterizing the symmetrized equilibrium noise,

$$\begin{pmatrix} s(\omega, T) \\ \tilde{s}(\omega, T) \end{pmatrix} = \begin{pmatrix} \Re g(\omega, T) \\ \Re \tilde{g}(\omega, T) \end{pmatrix} \frac{\omega}{T_K} \coth\left(\frac{\omega}{2T}\right) . \quad (27)$$

The structures of the universal functions g , \tilde{g} , s , and \tilde{s} shall be thoroughly discussed in the following Sections.

D. Non-collinearity

The discussion above can readily be generalized to the case, where the polarizations of the injected and detected currents are arbitrary, but the system still exhibits SU(2) symmetry. In this case we first define the charge of carriers polarized along \mathbf{n} as

$$Q_r^n \equiv e \sum_{\sigma, \sigma'} \int_{-D}^D c_{r\sigma}^\dagger(\varepsilon) \frac{1}{2} (\mathbf{1} + \mathbf{n} \cdot \boldsymbol{\sigma})_{\sigma\sigma'} c_{r\sigma'}(\varepsilon) d\varepsilon . \quad (28)$$

The corresponding current operators, $J_r^n = \frac{d}{dt} Q_r^n$ can be expressed as

$$J_r^n = I_r^n + \tilde{I}_r^n ,$$

the even and odd current components being defined as

$$\begin{aligned} I_r^n &= e j \gamma_r i (\Psi^\dagger P_n F - F^\dagger P_n \Psi) , \\ \tilde{I}_r^n &= e j \tilde{\gamma}_r i (\tilde{\Psi}^\dagger P_n F - F^\dagger P_n \tilde{\Psi}) , \end{aligned} \quad (29)$$

with the projector P_n given by

$$P_n = (\mathbf{1} + \mathbf{n} \cdot \boldsymbol{\sigma})/2 . \quad (30)$$

Apart from the above changes of definition, the calculations presented in Subsection II C trivially generalize to this case, and the final expression of the conductance assumes the following simple form,

$$\begin{aligned} G_{rr'}^{nn'}(\omega) &= \frac{e^2}{h} \left[\tilde{a}_{rr'} \frac{\mathbf{1} + \mathbf{n} \cdot \mathbf{n}'}{2} \tilde{g}(\omega, T) \right. \\ &\quad \left. + a_{rr'} \mathbf{n} \cdot \mathbf{n}' g(\omega, T) \right] , \end{aligned} \quad (31)$$

with the functions g and \tilde{g} defined by Eqs. (24). Through the fluctuation-dissipation theorem, we then obtain the following expression for the noise,

$$\begin{aligned} S_{rr'}^{nn'}(\omega) &= -\frac{e^2}{h} T_K \left[\tilde{a}_{rr'} \frac{\mathbf{1} + \mathbf{n} \cdot \mathbf{n}'}{2} \tilde{s}(\omega, T) \right. \\ &\quad \left. + a_{rr'} \mathbf{n} \cdot \mathbf{n}' s(\omega, T) \right] , \end{aligned} \quad (32)$$

with the functions s and \tilde{s} defined in Subsection II C. We emphasize again that the above expressions hold only in the presence of spin rotation invariance, and in an external magnetic field the conductance cannot be parametrized in terms of just two universal functions (see Sec. III B).

III. THE $T = 0$ TEMPERATURE LIMIT

Before discussing the more complex case of finite temperature, let us focus on the much simpler $T = 0$ case. There we can compute the universal scaling functions g and s "numerically exactly" using the machinery of numerical renormalization group (NRG),^{30,31} and can understand all their important features relatively easily by combining perturbative renormalization group methods with Fermi liquid arguments.

A. Analytical considerations

Perturbation theory. The simplest one can do to compute the noise is to evaluate the current-current correlation functions order by order in j . The 0-th order contribution to $S_{LR}^{\sigma\sigma'}$ is found to be

$$S_{LR}^{\uparrow\uparrow} = \frac{1}{2} S_{LR}^{\uparrow\downarrow} = -\frac{e^2}{h} \sin^2 \phi |\omega| \frac{\pi^2 j^2}{16} , \quad (33)$$

from which we can extract g and \tilde{g} using Eqs. (21) and (26),

$$\tilde{g} = \frac{3\pi^2 j^2}{16} + \dots ; \quad g = -\frac{\pi^2 j^2}{8} + \dots . \quad (34)$$

Evaluating higher order terms to $S_{LR}^{\sigma\sigma'}$, one obtains logarithmic corrections to g and \tilde{g} . The so-called leading logarithmic corrections can be summed up by a perturbative renormalization group procedure,^{32,33} which amounts in simply replacing $j \rightarrow j(\omega) = 1/\ln(\omega/T_K)$, and gives

$$\tilde{g} \approx \frac{3\pi^2}{16} \frac{1}{\ln^2(|\omega|/T_K)} ; \quad g \approx -\frac{\pi^2}{8} \frac{1}{\ln^2(|\omega|/T_K)} , \quad (35)$$

for large frequencies, $|\omega| \gg T_K$. By the fluctuation dissipation theorem, Eq. (27), we then find for $|\omega| \gg T_K$

$$\tilde{s} \approx \frac{3\pi^2}{16} \frac{|\omega|/T_K}{\ln^2(|\omega|/T_K)} ; \quad s \approx -\frac{\pi^2}{8} \frac{|\omega|/T_K}{\ln^2(|\omega|/T_K)} . \quad (36)$$

Fermi liquid regime. The above expressions approximate the scaling functions only at large frequencies, $|\omega| \gg T_K$. In the opposite limit, $|\omega| \ll T_K$, perturbation theory in j breaks down. However, the relevant processes can be captured by a simple Fermi liquid approach.²⁵ In this very small frequency limit both g and \tilde{g} are analytical. At $T = 0$ temperature the charge conductance is unitary in the limit, $\omega \rightarrow 0$, and correspondingly,

$$\tilde{g} = 1 + \mathcal{O}(\omega^2) , \quad \tilde{s}(\omega/T_K) = \frac{|\omega|}{T_K} + \dots . \quad (37)$$

In contrast to \tilde{g} , however, $G_{rr'}^{\uparrow\downarrow}(\omega \rightarrow 0)$ and thus $g(\omega \rightarrow 0)$ must vanish. This follows from the simple observation that it is impossible to generate a steady spin-down current in any of the leads by injecting only

spin-up electrons. This simply follows from Fermi liquid theory.²⁵ According to this latter, at $T = 0$ temperature the impurity spin is completely screened by the Kondo effect, and electrons injected at the Fermi energy are only subject to elastic scattering. As a consequence, a spin-up electron injected right at the Fermi energy conserves its spin. Electrons (quasiparticles), however, interact with each-other through a local interaction at the impurity site. For electrons injected with energy ω , this residual electron-electron interaction generates spin-flip processes, and leads to a finite spin cross-conductance. By simple phase space arguments, the amplitude of these latter processes scales with the square of the energy of the incoming electron, $\sim \omega^2$, and therefore $g(\omega) \sim \omega^2$. We thus conclude that

$$g = -\alpha \frac{\omega^2}{T_K^2} + \dots, \quad s = -\alpha \frac{|\omega|^3}{T_K^3} + \dots, \quad (38)$$

with α a universal parameter.

B. NRG results

In Section II B we related the functions g , and \tilde{g} to the correlation functions of the local operators, \mathcal{I}_σ and $\tilde{\mathcal{I}}_\sigma$, respectively [see Eq. (24)]. At $T = 0$ temperature, such local correlation functions can be accurately computed by NRG calculations,^{30,31} which, as we show below, are indeed in full agreement with our previous analytical considerations.

1. No external field, $B = 0$

Let us start by sketching how one can perform the NRG calculations in the case where we have no external magnetic field, \mathbf{B} . To compute \tilde{g} , we exploit the fact that correlation functions of the operators $\Psi_\sigma^\dagger F_\sigma$ and $F_\sigma^\dagger \tilde{\Psi}_\sigma$ can be factorized onto correlation functions of F_σ^\dagger and $\tilde{\Psi}_\sigma^\dagger$. The latter being trivial, after some algebra we then obtain for the real part of \tilde{g} ,

$$\Re \tilde{g}(\omega, T) = \frac{\pi^2 j^2}{4} \int \frac{d\omega'}{2\omega} \varrho_F(\omega', T) [f(\omega' - \omega) - f(\omega' + \omega)], \quad (39)$$

with $f(\omega)$ the Fermi function and $\varrho_F(\omega, T) \equiv -\frac{1}{\pi} \Im m (F_\uparrow^\dagger; F_\uparrow^\dagger)_\omega$ the spectral function of the composite fermion operator. The prefactor $\pi^2 j^2/4$ in Eq. (39) can be eliminated by observing that $\tilde{g}(0, 0) = 1$,²⁴ and therefore

$$\frac{\pi^2 j^2}{4} \varrho_F(0, 0) = 1. \quad (40)$$

Using this relation, we can also express the real part of the scaling function g as

$$\Re g(\omega, T) = -\frac{1}{2\omega} \frac{\varrho_{\mathcal{I}_\uparrow \mathcal{I}_\uparrow}(\omega, T)}{\varrho_F(0, 0)} \quad (41)$$

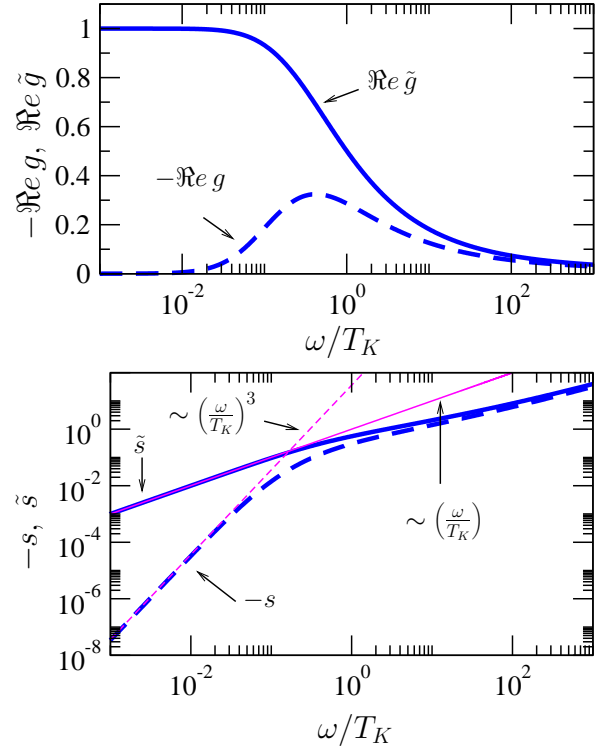


FIG. 6: (Color online) The zero temperature universal functions g , \tilde{g} for the a.c.-conductance (upper panel) and for the spin noise s and \tilde{s} (lower panel) computed by NRG.

with $\varrho_{\mathcal{I}_\uparrow \mathcal{I}_\uparrow}(\omega, T) = -\frac{1}{\pi} \Im m (\mathcal{I}_\uparrow; \mathcal{I}_\uparrow^\dagger)_\omega$ the spectral function of the operator, $\mathcal{I}_\sigma \equiv i(\Psi_\sigma^\dagger F_\sigma - F_\sigma^\dagger \Psi_\sigma)$. Computing thus the local spectral functions, ϱ_F and $\varrho_{\mathcal{I}_\uparrow \mathcal{I}_\uparrow}$ we can determine the real parts of the functions, g and \tilde{g} . From these, we can then directly determine the universal spin and charge current noise functions s and \tilde{s} .

The real parts of g and \tilde{g} and the corresponding scaling functions, s and \tilde{s} , as obtained by NRG⁵² are shown in Fig. 6. The results presented were obtained by the Budapest Flexible-DMNRG code,^{34,35} and clearly display all features discussed in the previous subsection. All functions were plotted as a function of ω/T_K , with T_K defined as a half-width of the function $\Re \tilde{g}$. This latter is essentially the a.c. charge conductance through the dot, studied in Ref. [36], and displays the expected Kondo resonance, which appears as a peak at frequencies $\omega < T_K$.

The behavior of the universal function, $-\Re g \sim G_{LR}^{\uparrow\downarrow}$, is somewhat more surprising. It exhibits a *maximum* at a frequency, $\omega \sim T_K$, in agreement with the results of our analytical considerations, Eqs. (35) and (38). This maximum corresponds to a temporary spin accumulation on the quantum dot at the “resonance” frequency, $\omega \sim T_K$.

The lower panel of Fig. 6 shows the universal functions for the spin noise on a logarithmic scale. The function \tilde{s} shows a clear linear behavior at frequencies $\omega < T_K$. The deviations at larger frequencies from the linear behavior

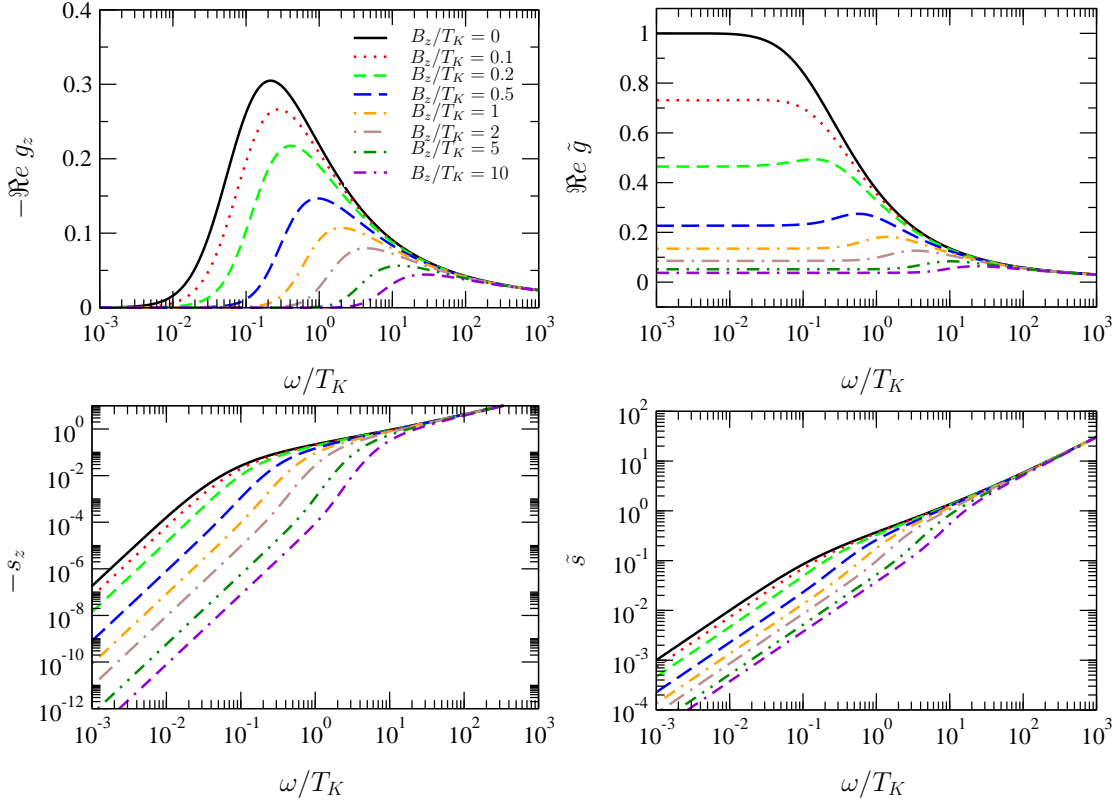


FIG. 7: (Color online) Real parts of the universal functions g_z and \tilde{g} , and the corresponding noise scaling functions, s_z and \tilde{s} , as obtained by performing zero-temperature NRG calculations with a magnetic field B_z along the z th direction.

are due to logarithmic corrections. At high frequencies $-s$ follows the behavior of \tilde{s} , and is linear apart from the aforementioned logarithmic corrections. For $\omega < T_K$, however, it deviates strongly, and scales to zero as $\sim \omega^3$.

2. Effect of Zeeman field, $B \neq 0$

The presence of a Zeeman field, $H_B = -\mathbf{B} \cdot \mathbf{S}$, breaks the spin SU(2) symmetry. Therefore, for general spin polarizations, \mathbf{n} and \mathbf{n}' , the spin current noise and the spin conductance cannot be characterized by just two universal functions, as before. Using group theoretical arguments and exploiting the electron-hole symmetry of the Kondo model we can show that 5 universal functions need be used to characterize the complete \mathbf{n} and \mathbf{n}' dependence of $G_{rr'}^{\mathbf{n}\mathbf{n}'}$. Moreover, these functions will not just be functions of ω/T_K and T/T_K , but they also depend on the ratio, B/T_K .

Here we do not venture to characterize all these functions. Rather, we focus our attention to the special case, where $\mathbf{n} \parallel \mathbf{n}'$. Without loss of generality, we can then assume that \mathbf{n} and \mathbf{n}' are both parallel to the z axis, $\mathbf{n} = \sigma \hat{z}$ and $\mathbf{n}' = \sigma' \hat{z}$. In this special case, one can show that, as a consequence of electron-hole symmetry and rotational invariance around the axis of the magnetic

field, the contribution of the current \tilde{I}_r^σ does not depend on the direction of the magnetic field, and that the total conductance has the structure,

$$G_{rr'}^{\sigma\sigma'}(\omega, \theta) = \frac{e^2}{h} \left\{ \delta_{\sigma\sigma'} \tilde{a}_{rr'} \tilde{g}(\omega, B) + \sigma\sigma' a_{rr'} [\cos^2(\theta) g_z(\omega, B) + \sin^2(\theta) g_\perp(\omega, B)] \right\}, \quad (42)$$

where θ is the angle between the magnetic field and \mathbf{n} , and two new scaling functions replace the original scaling function, $g(\omega, T)$. Notice that all these scaling functions now depend on the size of the magnetic field, too. Application of the fluctuation dissipation theorem yields then a similar scaling form for the noise, with corresponding scaling functions, \tilde{s} , s_z , and s_\perp ,

$$S_{rr'}^{\sigma\sigma'}(\omega, \theta) = -\frac{e^2}{h} T_K \left\{ \delta_{\sigma\sigma'} \tilde{a}_{rr'} \tilde{s}(\omega, B) + \sigma\sigma' a_{rr'} [\cos^2(\theta) s_z(\omega, B) + \sin^2(\theta) s_\perp(\omega, B)] \right\}. \quad (43)$$

To determine the scaling functions, \tilde{g} and g_z , and the corresponding noise scaling functions, we performed the zero-temperature NRG calculations for the case, where the magnetic field is parallel to the z axis, $B_z = B$ (i.e., $\theta = 0$). The results are summarized in Fig. 7. Application of a magnetic field gradually removes the spin degeneracy of the dot, and leads to a splitting and suppression

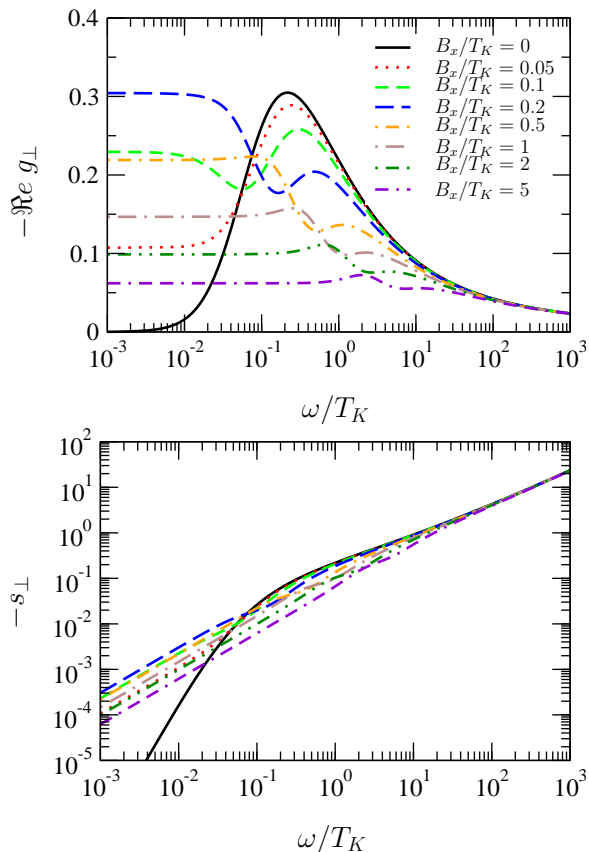


FIG. 8: (Color online) Zero temperature NRG results for the real part of the universal function g_{\perp} , and the corresponding noise scaling function s_{\perp} for several values of magnetic field pointing along the x direction.

of the Kondo resonance for $B \gg T_K$. The splitting of the resonance can readily be observed in the scaling function \tilde{g} (i.e., the charge conductance through the dot). At very large fields, $B \gg T_K$, the conductance has a logarithmic tail for $\omega \gg B$, but is strongly reduced at frequencies $\omega < B$, where spin-flip processes are forbidden.

Similar to g , the scaling function g_z continues to vanish at $\omega = 0$ for any magnetic field. This property follows from the overall conservation of the total spin component parallel to the external magnetic field. The only effect of a magnetic field is thus to increase the region of suppressed conductance.

Next, to determine the scaling function, g_{\perp} , we performed calculations with a field B_x applied along the x direction ($\theta = \pi/2$). We also checked that \tilde{g} is the same as before, and does not depend on the direction of the applied field. The function g_{\perp} , however, is markedly different from g_z . Of course, for $B = 0$ they are both equal to g . However, any finite magnetic field results in a finite d.c. conductance, $g_{\perp}(\omega = 0) \neq 0$. The reason of this is that in this case we measure a spin component which is not conserved for any finite magnetic field. In a sense, the perpendicular magnetic field in this case plays

a role similar to the external spin relaxation, discussed in Section VI, and allows the impurity spin to flip back and forth, independently of the conduction electrons. As a consequence, the "pseudogap" feature in g_{\perp} is gradually filled up with increasing B_x , reaching maximum for $B_x \sim T_K$.

IV. THE FINITE TEMPERATURE CASE

Although the NRG calculations presented in the previous section could, in principle be extended to any finite temperature, a serious *technical* problem appears. As we mentioned in the introduction, the cross-spin conductance, $G_{LR}^{\uparrow\downarrow}$ and thus g , must vanish at any finite temperature T in the $\omega \rightarrow 0$ limit. However, currently used finite temperature NRG broadening schemes all produce linear ($\propto \omega$) spectral weight in $\varrho_{T\uparrow T\uparrow}(\omega, T)$, and therefore by Eq. (41) lead to a finite and thus unphysical d.c. cross-spin conductance.

We do not know of any way to get around this problem. Therefore, at finite T , we had to rely on analytical results, and combine them with the $T = 0$ temperature results to obtain a coherent picture. At very high temperatures, $T \gg T_K$, we can make use of perturbative approaches. However, as explained below, even in this regime simple-minded perturbation theory is insufficient, and we need to combine it with a master equation approach to obtain the complete ω dependence of the spin conductance and noise. Combining these perturbative results with scaling and Fermi liquid arguments, we are then able to understand the complete frequency and temperature dependence of the scaling functions, s , \tilde{s} , g and \tilde{g} .

A. Perturbation theory and master equation approach

At temperatures $T \gg T_K$ corrections to the leading order perturbative results are small, and much of the noise spectrum can be understood based upon perturbative results. However, to understand the limitations of perturbation theory, we first need to understand the important time scales in this high temperature limit and the way they influence spin transport. At $T \gg T_K$, transport through the dot occurs through individual exchange processes, whereby just one electron tunnels from one side of the dot to the other side of it. The typical time between such events is given by the "Korringa time", τ_K , which we define as the inverse of the Korringa rate, $\tau_K \sim h/E_K$. To the leading order in j , it is given as, $\tau_K \sim h/j^2 T$. Tunneling events are, however, not instantaneous in the sense that they are dressed by the internal dynamics of the electron-hole excitations, created throughout the tunneling process. Correspondingly, the "duration" of a tunneling process is given by the thermal time, $\tau_T \equiv h/T \ll \tau_K$. At very short times below the thermal time, $t < h/T \equiv \tau_T$ (or at frequencies $\omega > T$),

current-current correlations reflect just the internal and coherent dynamics of such a spin-flip event, as well captured by the usual connected second order contribution to the current-current correlation function.

As just stated, the usual bubble diagram only accounts for the structure of a *single* tunneling event. At times $t > \tau_K$, however, several independent incoherent tunneling processes take place. These processes are correlated, since a spin-flip process that changes the dot spin from up into down ($\uparrow \rightarrow \downarrow$) must necessarily be followed by an opposite process when the dot spin flips from down to up ($\downarrow \rightarrow \uparrow$). These correlations turn out to be important for spin transport, and are obviously not captured by simple perturbation theory. Fortunately, for times $t > \tau_T$ ($\omega < T$), the internal dynamics of a spin-flip event can be ignored, and we can make use of a master equation method as a complementary approach. There tunneling processes are taken to be instantaneous, just characterized by some rates, but correlations between individual spin-flip events are properly accounted for through a classical rate equation.

From these simple arguments we thus conclude that the master equation approach must be valid for frequencies $\omega < T$, while simple-minded perturbation theory works for frequencies $E_K < \omega$. Since $E_K < T$, the range of validity of these two approaches overlaps, as also confirmed by the actual calculations presented below and by the results of Ref. [20].

1. Perturbation theory

As explained before, for $T \gg T_K$ and times $t < \tau_K$ (frequencies $\omega > E_K$), perturbation theory (PT) is thus a good approximation, though it fails at longer times where already several spin-flip events occur, and the correlations between these spin-flip events cannot be neglected. Simplest 0-th order perturbation theory yields, e.g., for the left-right components of the symmetrized frequency-dependent spin noise

$$\begin{aligned} S_{LR}^{\uparrow\downarrow}(\omega) &= -\frac{e^2}{h} \sin^2 \phi \frac{\pi^2 j^2}{8} \omega \coth\left(\frac{\omega}{2T}\right) + \dots, \\ S_{LR}^{\uparrow\uparrow}(\omega) &= -\frac{e^2}{h} \sin^2 \phi \frac{\pi^2 j^2}{16} \omega \coth\left(\frac{\omega}{2T}\right) + \dots, \end{aligned}$$

with the dots referring to higher order corrections in j . The corresponding universal functions then read for $\omega > E_K$,

$$\begin{aligned} s^{\text{PT}}(\omega) &= -\frac{2}{3} \tilde{s}^{\text{PT}}(\omega) = -\frac{\pi^2 j^2}{8} \frac{\omega}{T_K} \coth\left(\frac{\omega}{2T}\right) + \dots, \\ g^{\text{PT}}(\omega) &= -\frac{2}{3} \tilde{g}^{\text{PT}}(\omega) = -\frac{\pi^2 j^2}{8} + \dots. \end{aligned} \quad (44)$$

Higher order terms give logarithmic corrections, and lead to a renormalization of j in these expressions.

2. Master equation approach

Let us now focus on the "classical" frequency regime, $\omega < T$. Here we can use a simple master equation (ME) approach,³⁷ where we assume that, at any instance, the spin on the dot is either in a spin-up state $S = \uparrow$ or in a spin-down state $S = \downarrow$, with corresponding probabilities, $\mathcal{P}_S(t)$. Conduction through the dot and spin relaxation are generated by scattering events, q , generated by the exchange interaction, Eq. (9). These scattering events are taken to be instantaneous, and consist of the scattering of a spin σ electron from lead r to a spin σ' state in lead r' while changing the dot spin, $S \rightarrow S'$,

$$q \leftrightarrow \{r', \sigma', S' \leftarrow r, \sigma, S\}.$$

They occur with a rate, $\gamma(q) = \gamma_{r'\sigma' \leftarrow r\sigma}^{S' \leftarrow S} \propto j^2 v_r^2 v_{r'}^2$, following Fermi's golden rule.

The dynamics of the dot spin is described by a simple master equation,

$$\frac{d}{dt} \begin{pmatrix} \mathcal{P}_\uparrow \\ \mathcal{P}_\downarrow \end{pmatrix} = \begin{pmatrix} -\Gamma & \Gamma \\ \Gamma & -\Gamma \end{pmatrix} \begin{pmatrix} \mathcal{P}_\uparrow \\ \mathcal{P}_\downarrow \end{pmatrix}, \quad (45)$$

with the relaxation rate Γ given as $\Gamma \equiv \sum_{r,r',\sigma,\sigma'} \gamma_{r\sigma \leftarrow r'\sigma'}^{\uparrow \leftarrow \downarrow}$. From Eq. (45) it follows that spin polarization on the dot relaxes exponentially, $\langle S_z \rangle \sim e^{-2\Gamma t}$. Thus the rate Γ is related to the Korringa spin relaxation rate as, $E_{K,0} \equiv 2\Gamma$, which for the simple equilibrium case considered here takes on the usual expression,

$$E_{K,0} = 2\Gamma = \pi j^2 T. \quad (46)$$

Here the label "0" indicates that this is just the leading order expression of the Korringa rate, and higher order terms in perturbation theory renormalize it [see Eq. (1)].

To compute current-current correlations, we first notice that a scattering event q at some time τ induces current pulses in the leads, $J_r^\sigma(t) = \Delta Q_r^\sigma(q) \delta(t - \tau)$, with $\Delta Q_r^\sigma(q) \in \{\pm e, 0\}$ the amount of charge transferred. Similarly, a series of events, $\{\tau_n, q_n\}$ generates a current,

$$J_r^\sigma(t) = \sum_n \Delta Q_r^\sigma(q_n) \delta(t - \tau_n). \quad (47)$$

As a consequence, the stationary current-current correlation function can be simply expressed as

$$\langle J_r^\sigma(t) J_{r'}^{\sigma'}(0) \rangle = \delta(t) \sum_q P(q) \Delta Q_r^\sigma(q) \Delta Q_{r'}^{\sigma'}(q) + \sum_{q,q'} P(q,t; q',0) \Delta Q_r^\sigma(q) \Delta Q_{r'}^{\sigma'}(q'). \quad (48)$$

Here the first term describes the auto-correlation of individual scattering events, while the second term describes correlations between distinct tunneling events. The probability $P(q)$ in Eq. (48) denotes the stationary rate for a given type of event, $q = \{r', \sigma', S' \leftarrow r, \sigma, S\}$, and can be expressed as

$$P(q) = \gamma(q) \bar{\mathcal{P}}_S, \quad (49)$$

with $\bar{\mathcal{P}}_S$ the stationary probability distribution of the dot spin. The quantity $P(q, t; q', 0)$ denotes the joint probability rate of a scattering event q at time t and an event q' at time $t = 0$. For the events $q = \{r_2, \sigma_2, S_2 \leftarrow r_1, \sigma_1, S_1\}$ and $q' = \{r'_2, \sigma'_2, S'_2 \leftarrow r'_1, \sigma'_1, S'_1\}$ it can be expressed as

$$P(q, t; q', 0) = \gamma(q) \mathcal{P}_{S_1 \leftarrow S'_2}(t) \gamma(q') \bar{\mathcal{P}}_{S'_1}, \quad (50)$$

where $\mathcal{P}_{S_1 \leftarrow S'_2}(t)$ denotes the conditional probability that the dot spin evolves from state S'_2 to S_1 during time t . The function $\mathcal{P}_{S_1 \leftarrow S'_2}(t)$ is determined by the master equation, Eq. (45), and it is obviously this and only this quantity that generates time-dependent correlations between consecutive scattering events. Thus spin current correlations in this perturbative master equation approach are directly related to the time evolution of the dot spin.

Having set up this general framework, the detailed calculation of the classical noise spectrum is somewhat tedious, but straightforward. Therefore, instead of presenting further details on it, let us just continue with the discussion of the final results. Within the master equation approach, the left-right components of the spin noise read

$$S_{LR}^{\uparrow\downarrow}(\omega < T) \approx -\frac{e^2}{h} \frac{\pi}{4} \frac{E_{K,0}}{\omega^2 + E_{K,0}^2} \frac{\omega^2 \sin^2 \phi}{\omega^2 + E_{K,0}^2}, \quad (51)$$

$$S_{LR}^{\uparrow\uparrow}(\omega < T) \approx -\frac{e^2}{h} \frac{\pi}{8} \frac{E_{K,0}}{\omega^2 + E_{K,0}^2} \frac{(\omega^2 + 3 E_{K,0}^2) \sin^2 \phi}{\omega^2 + E_{K,0}^2}.$$

From these equations we extract the following approximations for the universal scaling functions,

$$s^{\text{ME}}(\omega) = -\frac{\pi^2}{4} j^2 \frac{T}{T_K} \frac{\omega^2}{\omega^2 + E_{K,0}^2}, \quad (52)$$

$$\tilde{s}^{\text{ME}}(\omega) = \frac{3\pi^2}{8} j^2 \frac{T}{T_K}. \quad (53)$$

Remarkably, for $T \gg \omega \gg E_{K,0}$, these results precisely coincide with the perturbative results, Eq. (44). This is indeed also clearly visible in Fig. 9, where we compare perturbation theory results for $S_{LR}^{\sigma\sigma'}(\omega)$ with results of

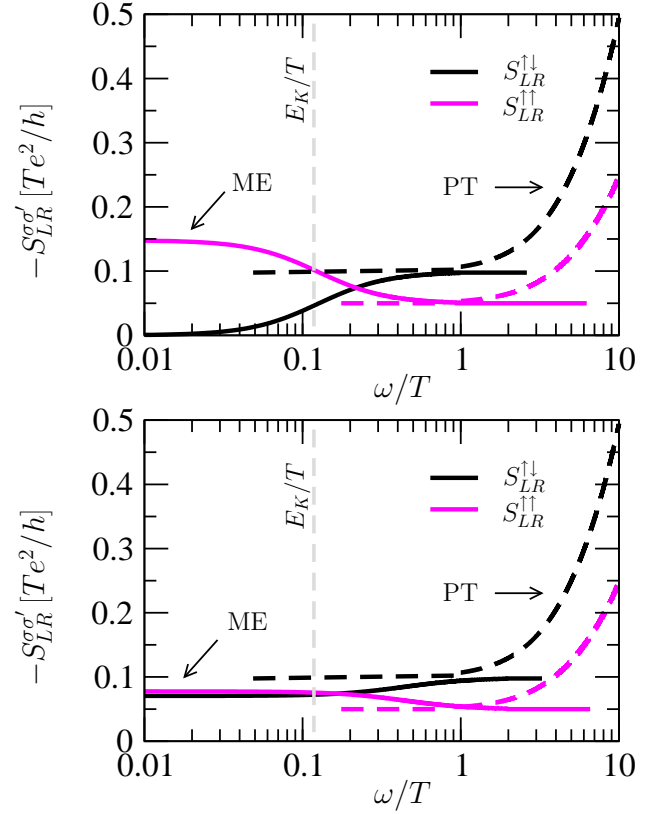


FIG. 9: (Color online) Upper panel: Frequency-dependent spin current cross-correlations ($S_{LR}^{\uparrow\downarrow}$) and auto-correlations ($S_{LR}^{\uparrow\uparrow}$) in the absence of spin relaxation, as calculated using the master equation approach (ME, solid line) and second-order perturbation theory (PT, dashed line). In the calculations we assumed $j = 0.2$. Lower panel: Effect of a finite (rather large) external spin relaxation rate, $1/\tau_s \equiv E_{K,0}/2$.

the master equation approach. We remark that one can bridge these two approaches through a systematic but much more formal and difficult quantum Langevin approach, already briefly sketched in Ref. [38], and to be discussed in a subsequent publication, Ref. [23].

The fluctuation dissipation theorem, Eq. (27), in this classical regime, $\omega < T$, amounts in the following expressions,

$$\Re g^{\text{ME}}(\omega) \approx \frac{T_K}{2T} s^{\text{ME}}(\omega) = -\frac{\pi^2}{8} j^2 \frac{\omega^2}{\omega^2 + E_{K,0}^2} \quad (54)$$

$$\Re \tilde{g}^{\text{ME}}(\omega) \approx \frac{T_K}{2T} \tilde{s}^{\text{ME}}(\omega) = \frac{3\pi^2}{16} j^2. \quad (55)$$

Notice that, in contrast to s and g , the functions \tilde{s} and \tilde{g} are completely featureless in this frequency range. On

the other hand, in agreement with our earlier statement, $S_{LR}^{\uparrow\downarrow}(\omega)$, $\Re G_{LR}^{\uparrow\downarrow}(\omega)$, s and $\Re g$ all exhibit a dip below E_K , and scale to zero as $\omega \rightarrow 0$. There is a simple heuristic picture behind this fact: a spin-flip process $\uparrow \rightarrow \downarrow$ pumps spin-up electrons into the leads. However, it must necessarily be followed by a reverse spin-flip process, $\downarrow \rightarrow \uparrow$, where, on average, exactly the same amount of spin is pumped back as it has been pumped in before. These processes exactly balance each other in the long time limit, and lead to a vanishing cross-spin conductance in equilibrium. We remark that if, however, there are external spin relaxation processes, then the spin may flip back spontaneously before pumping back the injected spin through the reverse spin-flip process. In this case, as we shall see in Sec. VI, the conductance $G_{LR}^{\uparrow\downarrow}(\omega)$ remains finite even in the $\omega \rightarrow 0$ limit (see also Fig. 9).

A rather curious consequence of the dip in s is that, while $S_{LR}^{\uparrow\downarrow}(\omega)$ develops a *dip* below the Korringa rate, the noise component $S_{LR}^{\uparrow\uparrow}(\omega)$ exhibits a *peak* of equal size, which precisely cancels the dip of $-S_{LR}^{\uparrow\downarrow}(\omega)$ in the charge noise. This peak in $S_{LR}^{\uparrow\uparrow}(\omega)$ or the similar peak in $S_{LL}^{\uparrow\uparrow}(\omega)$ may be more conveniently detected experimentally than cross-spin correlations.

B. Beyond perturbation theory

1. Logarithmic corrections

In the previous subsection we discussed only the leading order perturbative and master equation results. Performing, however, perturbation theory in j gives rise to logarithmic corrections. As long as $\max\{T, |\omega|\} \gg T_K$, these corrections can be summed up using renormalization group methods,^{32,33} and amount in the replacement of j by its renormalized value, $j \rightarrow 1/\ln(\max\{T, |\omega|\}/T_K)$. Apart from this substitution, however, the results of Subsections IV A 1 and IV A 2 continue to be valid as long as $T \gg T_K$. For $\omega > T$, e.g., we just recover the $T = 0$ temperature results, Eqs. (35) and (36), while in the opposite limit, $\omega < T$, we obtain

$$\begin{aligned} s(\omega) &\approx -\frac{\pi^2}{4} \frac{1}{\ln^2(T/T_K)} \frac{T}{T_K} \frac{\omega^2}{\omega^2 + E_K^2(T)}, \\ \tilde{s}(\omega) &\approx \frac{3\pi^2}{8} \frac{1}{\ln^2(T/T_K)} \frac{T}{T_K}, \end{aligned} \quad (56)$$

with $E_K = E_K(T) = \pi T / \ln^2(T/T_K)$ the renormalized Korringa rate of Eq. (1). Similarly, for the scaling functions g and \tilde{g} we obtain in this regime,

$$\begin{aligned} \Re g(\omega) &\approx \frac{T_K}{2T} s(\omega) = -\frac{\pi^2}{8} \frac{1}{\ln^2(T/T_K)} \frac{\omega^2}{\omega^2 + E_K^2(T)}, \\ \Re \tilde{g}(\omega) &\approx \frac{T_K}{2T} \tilde{s}(\omega) = \frac{3\pi^2}{16} \frac{1}{\ln^2(T/T_K)}. \end{aligned} \quad (57)$$

Fig. 10 gives a concise summary of these results.

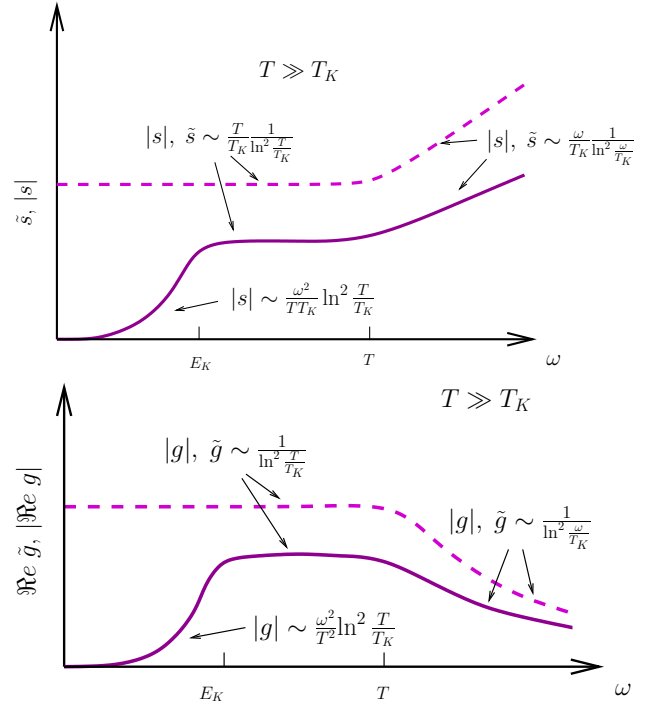


FIG. 10: (Color online) Sketch of the universal scaling functions s (continuous line), and \tilde{s} (dashed line), and the real parts of g (continuous line) and $\tilde{g}(\omega/T_K, T/T_K)$ (dashed line) for $T \gg T_K$. The charge conductance and noise functions, \tilde{g} and \tilde{s} show no particular feature, while the spin scaling functions exhibit an anomaly below the Korringa rate, E_K .

2. Fermi liquid regime, $T \ll T_K$

In the Fermi liquid regime,²⁵ $T \ll T_K$, perturbation theory in j breaks down. However, we can derive the behavior of the scaling functions by two simple observations. We first observe that in this Fermi liquid regime, $j \rightarrow \infty$,^{25,30} and therefore the only remaining energy scales are T and T_K . Our second observation is that at the Fermi liquid fixed point, the residual electron-electron interactions are irrelevant,²⁵ and therefore physical quantities are analytical functions of ω . In particular, the asymptotic forms, Eqs. (37) and (38) remain valid up to the higher order corrections even at finite temperatures,

$$\Re g(\omega) = -\alpha \frac{\omega^2}{T_K^2} + \mathcal{O}(\omega^4, T^2 \omega^2), \quad (58)$$

$$\Re \tilde{g}(\omega) = 1 + \mathcal{O}(\omega^2, T^2). \quad (59)$$

The scaling functions of the noise, s and \tilde{s} can then yet again be read out of the fluctuation dissipation theorem, Eq. (27), yielding

$$s(\omega \ll T_K) \approx -\alpha \frac{\omega^3}{T_K^3} \coth\left(\frac{\omega}{2T}\right), \quad (60)$$

$$\tilde{s}(\omega \ll T_K) \approx \frac{\omega}{T_K} \coth\left(\frac{\omega}{2T}\right). \quad (61)$$

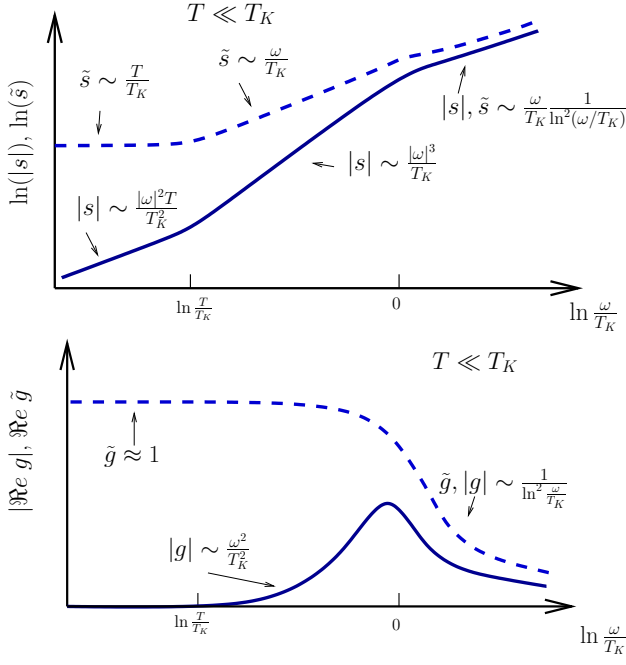


FIG. 11: (Color online) Sketch of the universal scaling functions s (continuous line) and \tilde{s} (dashed line) and the real parts of g (continuous line) and \tilde{g} (dashed line) for $T \ll T_K$.

These equations reduce to the $T = 0$ expressions in the $\omega \gg T$ limit, and are valid as long as $\omega < T_K$. Notice that in the $\omega \rightarrow 0$ limit, the charge noise component \tilde{s} scales to a constant, $\propto T$, while the spin component s scales quadratically to zero, just as in the $T \gg T_K$ regime. Of course, for $\omega > T_K$ the scaling functions must also reduce to their $T = 0$ temperature expressions, Eqs. (35) and (36). The overall behavior of these functions for $T \ll T_K$ is sketched in Fig. 11

V. TRANSIENT RESPONSE

Let us now turn to the discussion of real time transient response, i.e., the time dependent current response in the left lead when a spin-dependent voltage of the form $V_R^\uparrow(t) = \delta V_R^\uparrow \theta(t)$ is applied to the right electrode (see Fig. 1). Within linear response theory, the average current pulse is just given by

$$\langle J_L^\sigma(t) \rangle = i \int_{-\infty}^{\infty} G_{LR}^{\sigma\uparrow}(\omega) \frac{1}{\omega - i\delta} e^{-i\omega t} d\omega \delta V_R^\uparrow, \quad (62)$$

with $i/(\omega - i\delta)$ the Fourier transform of the θ function.

Surprisingly, just using Eq. (62) and the analytical properties of the functions $G_{LR}^{\sigma\uparrow}(\omega)$, we are able to make rather strong statements on the transient response, $\langle J_L^\sigma(t) \rangle$. Let us start by briefly summarizing these analytical properties. First of all, being retarded response functions, $G_{LR}^{\sigma\sigma'}(\omega)$, are analytical on the upper half plane. Moreover, as assured by Fermi liquid theory, they are

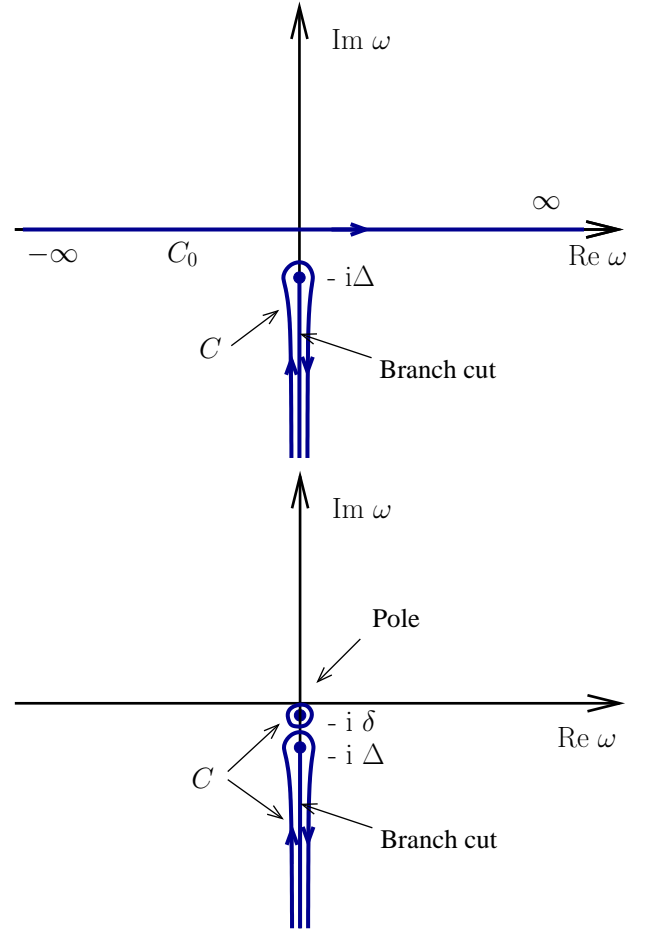


FIG. 12: (Color online). Pole structure of the integrand in Eq. (62), for the spin-up - spin-down (upper panel) and spin-up - spin-down (lower panel) channels. In the spin- $\uparrow\downarrow$ channel, the pole at $\omega = 0$ is canceled by the ω^2 dependence of $g(\omega, T)$, while in the case of spin- $\uparrow\uparrow$ this pole survives and gives a finite response as $t \rightarrow \infty$.

also analytical in an extended region around $\omega = 0$ at any temperature. However, from perturbation theory we know that at very large frequencies, $\omega \gg T_K, T$, they have logarithmic tails, $|G_{LR}^{\sigma\sigma'}(\omega)| \sim 1/\ln^2(\omega/T_K)$ and thus tend to zero even in the universal scaling limit, $D \rightarrow \infty$, T_K finite. Their asymptotic behavior and their symmetries [$\text{Re } G_{rr'}^{\sigma\sigma'}(\omega) = \text{Re } G_{rr'}^{\sigma\sigma'}(-\omega)$ while $\text{Im } G_{rr'}^{\sigma\sigma'}(\omega) = -\text{Im } G_{rr'}^{\sigma\sigma'}(-\omega)$] imply the presence of a cut along the negative imaginary axis with an endpoint, $-i\Delta$, with $\Delta \propto \max\{T, T_K\}$ (see Fig. 12).⁵³ Furthermore, as already discussed, $G_{rr'}^{\uparrow\downarrow}(\omega = 0) = 0$, while the components $G_{rr'}^{\uparrow\uparrow}(\omega = 0) = G_{rr'}^{\downarrow\downarrow}(\omega = 0)/2$ remain finite.

Let us now discuss the properties of the response, Eq. (62). First, we notice that due to the asymptotic $1/\ln^2(\omega/T_K)$ fall-off of $G_{LR}^{\sigma\sigma'}(\omega)$ and the analyticity on the upper half-plane, the integral contours in Eq. (62) can be closed upwards for any time $t \leq 0$. Therefore, $\langle J_L^\sigma(t) \rangle = 0$ for $t \leq 0$, i.e., it respects causality. The

response being zero even at $t = 0$ is not entirely trivial: in the master equation approach, e.g., $G_{rr'}^{\uparrow\uparrow}(\omega)$ remains finite in the $\omega \rightarrow \infty$ limit, and one obtains an unphysical jump at $t = 0$. We notice that the statement on the $t = 0$ response being zero is equivalent to the Kramers-Kronig relation. Though the response $\langle J_L^\sigma(t) \rangle$ vanishes at time $t = 0$ and is continuous for times $t \geq 0$, the *slope* of the response, $\frac{d}{dt} \langle J_L^\sigma(t) \rangle|_{t=0}$ is, however, infinite, since the integral $\int_{-\infty}^{\infty} d\omega G_{rr'}^{\uparrow\uparrow}(\omega)$ logarithmically diverges.

For times $t > 0$, the contours must be closed downwards, as shown in Fig. 12. In the spin-up – spin-down channel, $G_{LR}^{\uparrow\downarrow}(\omega = 0) = 0$, and therefore the pole at $-i\delta$ does not give any contribution. The contribution of the cut to the spin up-down response can be written as

$$\begin{aligned} \frac{\langle J_L^\downarrow(t) \rangle}{\delta V_R^\uparrow} &= - \int_C G_{LR}^{\uparrow\downarrow}(z) \frac{1}{z} e^{-izt} dz \\ &= \frac{e^2}{h} \sin^2 \phi e^{-\Delta t} \int_0^\infty \delta g(\Delta + y) e^{-yt} \frac{dy}{y}, \end{aligned} \quad (63)$$

with $\delta g(x) = 2 \Im m g(-ix + \delta)$ the cut of the universal conductance function, $g(\omega)$. Clearly, the contribution of the cut falls off as $\sim e^{-\Delta t}$ for long times. At $T = 0$ temperature we have $\Delta \propto T_K$, and furthermore δg must be also a universal function, $\delta g(\Delta + y) = \delta g(y/T_K)$. Therefore, the response is a universal function of tT_K . We can also tell the short time asymptotics of the response. Making use of the fact that the response is continuous at $t = 0$, we obtain for $t \ll 1/\Delta$ the expression,

$$\langle J_L^\downarrow(t) \rangle \sim \frac{e^2}{h} \sin^2 \phi \int_0^\infty \delta g(\Delta + y) (e^{-yt} - 1) \frac{dy}{y}. \quad (64)$$

Since the cut scales for large energies as $\sim 1/\ln^3(y/T_K)$, we get,

$$\langle J_L^\downarrow(t \ll 1/\Delta) \rangle \sim \frac{\delta V_R^\uparrow}{\ln^2 \frac{1}{tT_K}}. \quad (65)$$

Remarkably, this result does not depend on the temperature, since it is determined only by the high frequency part of the conductance. Furthermore, since the length of the current pulse is determined by the exponential prefactor, $\sim e^{-\Delta t}$, we can also read out of Eq. (65) its height: for $T \ll T_K$ one has $\Delta \sim T_K$, and the height of the pulse is $\langle J_L^\downarrow(t) \rangle \sim \delta V_R^\uparrow$. For $T \gg T_K$, on the other hand, we have $\Delta \sim T$, and the height of the current pulse, is $\sim \delta V_R^\uparrow / \ln^2 \frac{T}{T_K}$.

Figure 13 summarizes all the characteristic features of the current response $\langle J_L^\downarrow(t) \rangle$, discussed above. The total charge pumped into spin-down channel of left lead is simply given by the integral of the transient response, and is approximately

$$\Delta Q_L^\downarrow \sim \frac{e^2}{h} \sin^2 \phi \begin{cases} \frac{1}{T \ln^2(T/T_K)}, & \text{if } T \ll T_K, \\ \frac{1}{T_K}, & \text{if } T \gg T_K. \end{cases} \quad (66)$$

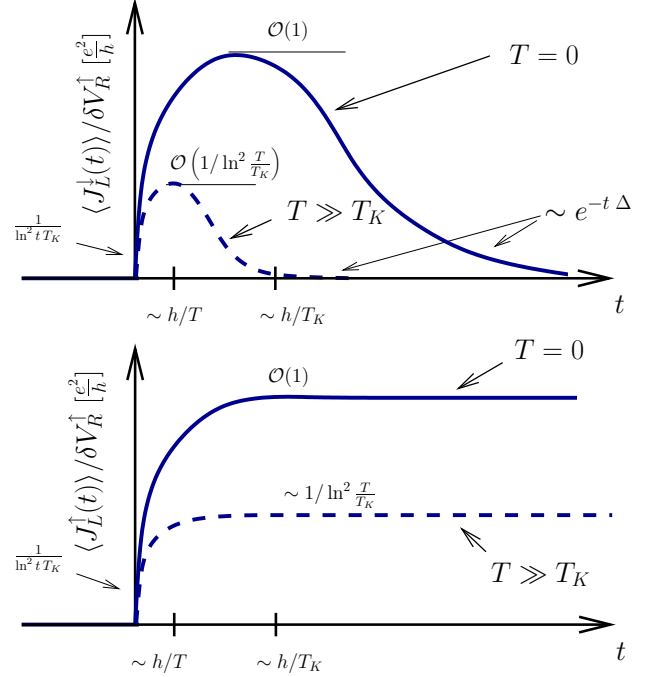


FIG. 13: (Color online). Transient current response in the spin-down (upper panel) and spin-up (lower panel) channels, upon a constant bias applied at $t = 0$ in the spin-up channel.

Remarkably, the coefficients appearing in this expression are just the high-temperature and low temperature expressions of the spin susceptibility.³⁹

The analysis of the response of the spin-up carriers follows very similar lines. The only major difference is that in this case the pole at $-i\delta$ gives a finite time independent contribution, and leads to an asymptotic response,

$$\langle J_L^\uparrow(t \rightarrow \infty) \rangle = G_{LR}^{\uparrow\uparrow}(\omega = 0) \delta V_R^\uparrow. \quad (67)$$

Otherwise, our discussions on the universal form of the response, and its short time $1/\ln^2(t)$ singularity carry over to this case, too. Instead of giving further details on $\langle J_L^\uparrow(t \rightarrow \infty) \rangle$, we just summarized its properties in Fig. 13.

VI. SPIN RELAXATION EFFECTS

All results presented so far were obtained under the assumption that spin relaxation is generated by the exchange coupling j , and there are no external sources of spin relaxation. In reality, however, external spin relaxation channels are always present. In quantum dots, the dominant channel of (external) spin relaxation is usually due to hyperfine interaction between the confined electron and nuclear spins in the host material, leading typically to a dephasing time of the order of $\tau_s \sim 10$ ns or longer in the absence of magnetic field.^{40,41} These hyperfine relaxation processes are thus characterized by an energy scale $h/\tau_s \sim 1 - 10$ mK, typically much smaller than

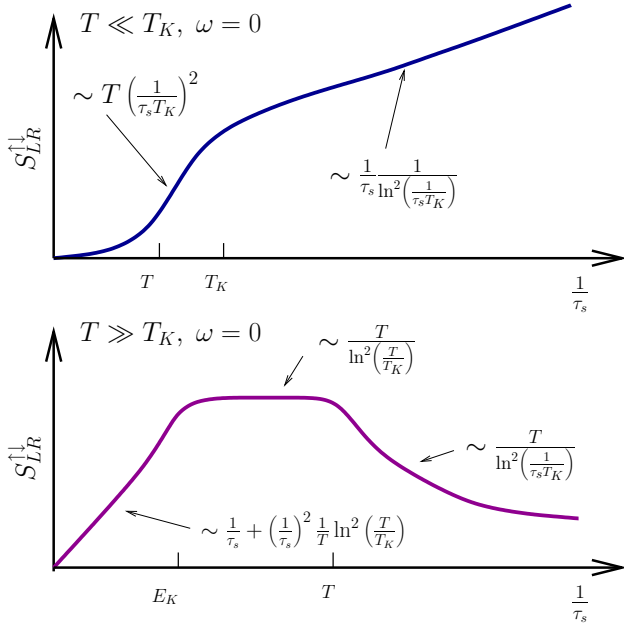


FIG. 14: (Color online) Shot noise $S_{LR}^{\uparrow\downarrow}(\omega=0, T)$ as a function of the spin relaxation rate $1/\tau_s$ in the Fermi liquid regime ($T \ll T_K$) (upper panel) and in the perturbative regime ($T \gg T_K$) (lower panel).

the temperature. Coupling to piezoelectric phonons⁴² or electromagnetic fluctuations⁴³ through spin-orbit interaction or polaron dephasing processes due to coherent acoustic phonon generation are, in general, characterized by even longer dephasing times and smaller relaxation rates.⁴⁴ Therefore, for typical experimental parameters, we would naively expect $1/\tau_s$ to be small compared to the temperature, T . Nevertheless, a finite τ_s leads to qualitatively different results, since its presence lifts the constraint of spin conservation, and allows to have a finite d.c. spin cross-conductance, $G_{LR}^{\uparrow\downarrow}(\omega=0) \neq 0$.

To investigate the effect of a finite τ_s , let us consider the perturbative ($T \gg T_K$) and Fermi liquid ($T \ll T_K$) regimes separately. In the regime $T \gg T_K$, we can readily extend our master equation analysis to include $1/\tau_s$ and obtain,

$$S_{LR}^{\uparrow\downarrow}(\omega < T) \simeq -\frac{e^2}{h} \frac{\pi}{4} \frac{E_K}{\omega^2 + E_K + \frac{1}{\tau_s^2}} \left(\omega^2 + \frac{1}{\tau_s^2} + \frac{E_K}{\tau_s} \right) \sin^2 \phi.$$

Here we incorporated already logarithmic corrections by replacing the bare Korringa rate, $E_{K,0}$ by its renormalized value, given by Eq. (1). Clearly, at large frequencies, $\omega \gg E_K$, external spin relaxation does not play a role. However, for $\omega < E_K$, it suppresses the dip in $S_{LR}^{\uparrow\downarrow}(\omega)$, and leads to a finite cross-spin "shot noise",

$$\begin{aligned} S_{LR}^{\uparrow\downarrow}(\omega=0) &\approx -\frac{e^2}{h} \sin^2 \phi \frac{\pi}{4} \frac{E_K}{E_K + \frac{1}{\tau_s}} \\ &\approx -\sin^2 \phi \frac{e^2}{h} \frac{\pi}{4} \min\{E_K, \frac{1}{\tau_s}\}. \end{aligned} \quad (68)$$

In other words, in the most relevant case, $1/\tau_s < E_K$, the cross-spin current noise is simply proportional to $1/\tau_s$, as also found by Kindermann.²⁰ The full frequency spectrum of the noise in the perturbative regime ($T \gg T_K$) is presented in Fig. 9, lower panel.

In the Fermi liquid regime, $T \ll T_K$, finite external spin relaxation also leads to a finite cross-spin noise. To estimate it, we first notice that $1/\tau_s$ just introduces a new frequency scale, and therefore we expect

$$G_{LR}^{\uparrow\downarrow}(\omega \rightarrow 0) \sim \frac{1}{\tau_s^2 T_K^2}. \quad (69)$$

Notice that T does not appear in this equation. Correspondingly, the noise behaves for $T < T_K$ as

$$S_{LR}^{\uparrow\downarrow}(\omega \rightarrow 0) \sim \frac{T}{\tau_s^2 T_K^2}. \quad (70)$$

The overall dependence of the cross-spin shot noise signal on the spin relaxation rate is sketched in Fig. 14. There we also display the physically not too relevant regime, $1/\tau_s > T$, where $1/\tau_s$ becomes the dominant energy scale, and therefore we have $j \rightarrow 1/\ln(1/\tau_s T_K)$.

VII. CONCLUSIONS

In the present paper we studied the equilibrium spin current noise and spin conductance through a quantum dot in its Kondo (local moment) regime. We have shown that in the absence of external fields, they are both characterized by a pair of universal functions, and determined the properties of these functions. We have shown that – in contrast to the charge conductance (G_{LR}) – the d.c. spin cross-conductance ($G_{LR}^{\uparrow\downarrow}$) vanishes. Put in another way, there is no spin drag, and a spin-up current cannot generate a steady spin-down current, at least not within linear response. At $T = 0$ temperature this obviously follows from Fermi liquid properties, and is thus valid for *any* interacting system with no spin-orbit coupling and with a Fermi liquid ground state. However, somewhat surprisingly, though it is not true for any interacting system, for a quantum dot, this property also carries over for finite temperatures. It is related to the simple structure of the Kondo (or the underlying Anderson) models, where spin transfer between spin-up and spin-down states can occur only through a single point, namely the dot state (or the dot spin in the Kondo model). Therefore, the spin currents generated by consecutive $\uparrow \rightarrow \downarrow$ and $\downarrow \rightarrow \uparrow$ flips of the dot spin precisely cancel each-other, and no d.c. cross-spin currents appear. Correspondingly, the cross-spin shot noise, $S_{LR}^{\uparrow\downarrow}(\omega=0, T)$ also vanishes at *any* temperature, and the noise spectra, $S_{LR}^{\uparrow\downarrow}(\omega)$ and $S_{LR}^{\uparrow\uparrow}(\omega)$ both exhibit related low frequency anomalies.

External spin relaxation slightly changes the picture above. It partly removes the correlations between consecutive spin-flip processes, and makes $G_{LR}^{\uparrow\downarrow}(\omega=0)$ and

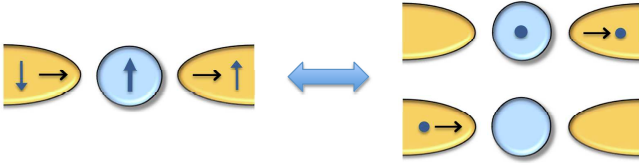


FIG. 15: (Color online) Correspondence between a single quantum dot device and a spinless double dot device.

$S_{LR}^{\uparrow\downarrow}(\omega = 0)$ finite. However, since the external spin-flip rate, $1/\tau_s$ is typically much smaller than the other energy scales (T , T_K , E_K), it only leads to small changes in the overall behavior of the noise and conductance functions.

As we also demonstrated in detail, simple-minded perturbation theory accounts only for the structure of individual coherent processes, and fails badly to capture these correlations between consecutive processes, which happen to dominate the spin response at small frequencies. Therefore, one must be very careful when calculating spin transport properties. Even in the perturbative regime, $T \gg T_K$, simple-minded perturbation theory is valid only for frequencies above the Korrington rate, $\omega > E_K$. To capture the physics at frequencies $\omega < E_K$, a supplementary master equation approach (valid for $\omega < T$) can be employed. Alternatively, one can use a more systematic but also more technical quantum Langevin approach, which works for any frequency in the perturbative regime, $T \gg T_K$, but neglects logarithmic corrections (see Ref. [23,45]).

Although finite-frequency noise measurements are now

available,^{12–17} and spin polarized currents can also be relatively easily produced,^{6,46} measuring the low-frequency anomalies of spin cross-correlations, $S_{LR}^{\uparrow\downarrow}(\omega)$, seems to be a difficult task. However, the predicted low frequency anomalies are also present in the spin polarized conductance, $G_{LR}^{\uparrow\uparrow}(\omega)$, and noise, $S_{LR}^{\uparrow\uparrow}(\omega)$ (see Figs. 3 and 5). These are experimentally much more easily accessible, since carriers must be polarized in the same direction. Alternatively, one can measure these cross-correlations in the *charge sector*, by using capacitively coupled double dots (see Fig. 15).^{47,48} In the spin polarized case, the Hamiltonian of the double dot system maps to that of the Anderson model with anisotropic hybridization parameters. Measuring noise or conductance between leads attached to the upper or lower leads of the double dot device shown in Fig. 15 is thus equivalent to cross-spin measurements in the single quantum dot setup.

Acknowledgments

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- ⁴⁹ Although in this paper we use units of $\hbar = k_B = 1$, in certain formulas we shall restore and display the Planck constant to clarify physical dimensions.
- ⁵⁰ The density of states is incorporated in the fields, $c_{r\sigma}(\varepsilon)$, to obtain the normalization in the main text.
- ⁵¹ The couplings v_L and v_R can be taken to be real.
- ⁵² In NRG calculations we kept 1024 states at each iteration and assumed $\Lambda = 1.8$ and $j = 0.18$.
- ⁵³ We remark that while perturbation theory indeed reproduces this cut, the master equation approach fails to do that, and only produces a pole at $-iE_K$.